October Math Gems
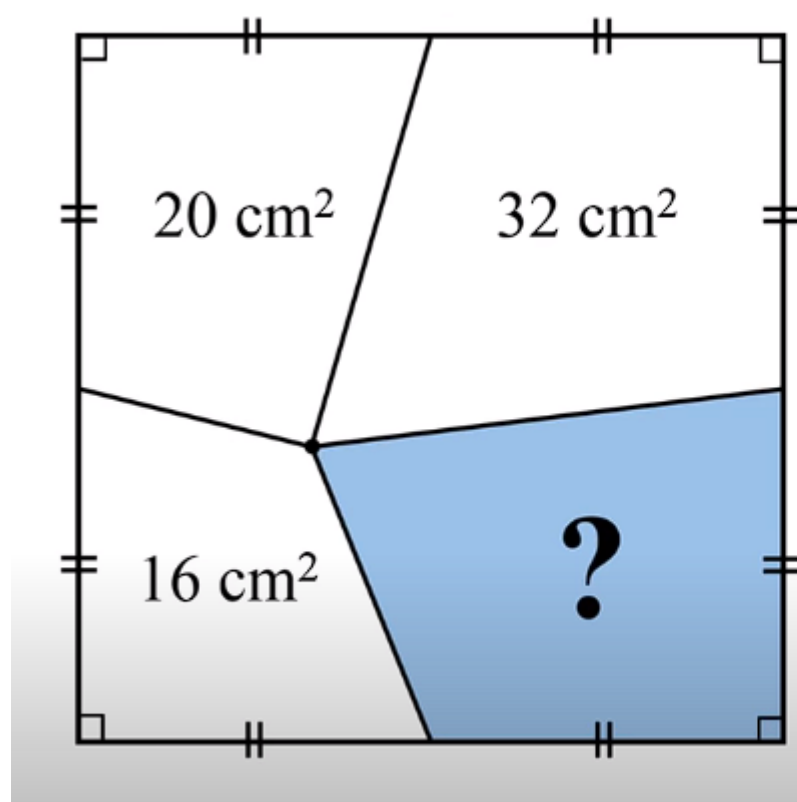
§1 Problems

Problem 1.1. What is the length of $|XY|$

(Diagram not to scal[e]

Problem 1.2. Suppose that α, β, and γ are the roots of the equation $x^3+3x^2-24x+1=0$. Find the value $\sqrt[3]{\alpha} + \sqrt[3]{\beta} + \sqrt[3]{\gamma}$

Problem 1.3. Find all real solution for the eqaution $\sqrt[3]{x} + \sqrt[3]{x-16} = \sqrt[3]{x-8}$

Problem 1.4. Find $\frac{a}{b}$
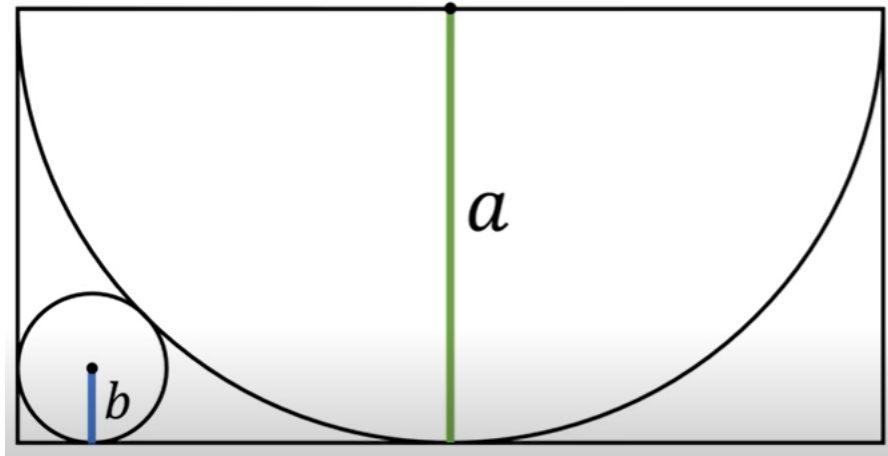
Problem 1.5. In the equation $ax^2 + 5x + 2$, solve for a so that the equation has exactly one solution

Problem 1.6. Given that $x^2 = 17x + y$ and $y^2 = 17y + x$. Then, what is the value of $\sqrt{x^2 + y^2 + 1}$?

Problem 1.7. Given that $\frac{1}{\sqrt[3]{25} + \sqrt[3]{5} + 1} = A\sqrt[3]{25} + B\sqrt[3]{5} + C$. So, what is the value of $A + B + C$ (A,B, and C are rational numbers).

Problem 1.8. Solve for all real values of x

$$4^x + 6^x = 9^x$$

Problem 1.9. $sin^2\,(1°\,) + sin^2\,(2°\,) + sin^2\,(3°\,) + \cdots\cdots + sin^2\,(90°\,)$

Problem 1.10. If $x = \sqrt{3\sqrt{2\sqrt{3\sqrt{2\sqrt{3\sqrt{2...}}}}}}$, Find the value of x^2

Problem 1.11. Given that $4x^2 + \frac{1}{x^2} = 2$. Then, what is the value of $8x^3 + \frac{1}{x^3}$?

Problem 1.12. The leg of a right-angled triangle is equal to $\frac{1}{5}$ the sum of the other sides, and the triangle's perimeter is 1. What is the area of the triangle?

Problem 1.13. Find all positive integers for which $n^2 + 45$ is a perfect square

Problem 1.14. A club has 30 members. The positions of president, vice president, and treasurer will be assigned to 3 distinct members. What is the maximum number of distinct assignments that can be made?

Problem 1.15. There are 90 equally spaced dots marked on a circle. Shannon chooses an integer, n. Beginning at a randomly chosen dot, Shannon goes around the circle clockwise and colours in every nth dot. He continues going around and around the circle colouring in every nth dot, counting each dot whether it is coloured in or not, until he has coloured in every dot. Which of the following could have been Shannon's integer ? $(3, 5, 6, 7)$

Problem 1.16. What is the value of x?
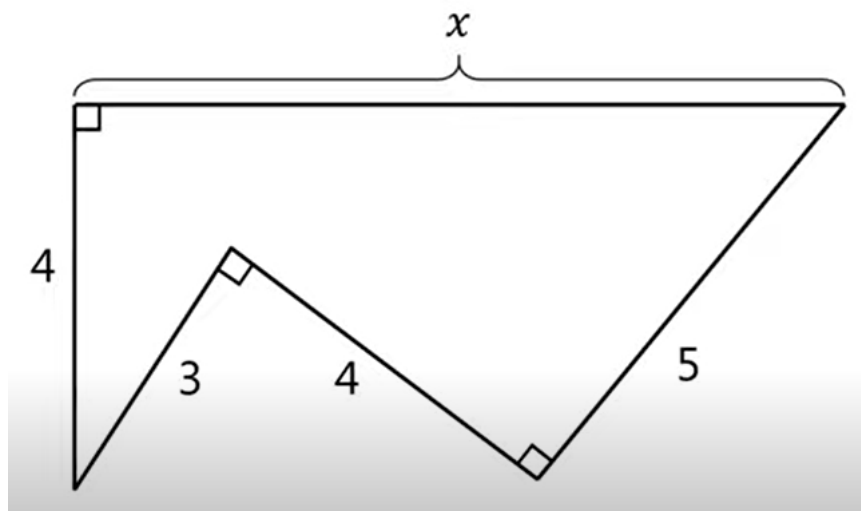
Problem 1.17. If $x^2 = 2023 + y$, $y^2 = 2023 + x$, where $x \neq y$ and x and y are both real numbers. Find the value of xy

Problem 1.18. Solve for real values of x

$$2\sqrt[3]{2x + 1} = x^3 - 1$$

Problem 1.19. Simplify and prove your answer for real values of x

$$x = \sqrt[3]{8 + 3\sqrt{21}} + \sqrt[3]{8 - 3\sqrt{21}}$$

Problem 1.20. Solve for all real solutions of x

$$(x^2 - 7x + 11)^{(x^2 - 13x + 42)} = 1$$